\( \Sigma^0 - \Lambda^0 \) Relative Parity from \( \Sigma^0 \) Decay

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In order to establish how \( \varepsilon \), the \( \Sigma^0 - \Lambda^0 \) relative parity, can be measured from actual bubble-chamber experiments featuring polarized \( \Sigma^0 \) production and decay, followed by \( \Lambda^0 \) decay and \( \gamma \)-pair production or Dalitz pair in the \( \Sigma^0 \) decay, we constructed a correlation function depending on \( \varepsilon \), another unknown parameter to be measured in the same experiment, and the energy and momenta of the different particles involved. Our study is Lorentz covariant, but the link with the usual "nonrelativistic" formalism is exhibited. In an Appendix it is shown that the polarization of \( \Sigma^0 \) produced in \( \pi^- + p^+ \) reactions is expected to be large.

**INTRODUCTION**

Since the \( \Sigma^0 \) decay is not due to weak coupling, it very likely conserves parity and it can be a tool to measure the \( \Sigma^0 - \Lambda^0 \) relative parity. This has been proposed by several authors\(^5\) who have shown the existence of two different relations between the three particle polarizations depending on the sign of \( \varepsilon \), the \( \Sigma^0 - \Lambda^0 \) relative parity.

The aim of this work is to show how \( \varepsilon \) can effectively be measured from an actual bubble-chamber experiment. For this we shall construct a correlation function whose variables are: \( \varepsilon \), another unknown parameter denoted by \( \varepsilon' \) (such that \(-1 \leq \varepsilon' \leq 1\) which is to be determined by the same experiment, and the energy and momenta of the different involved particles.

Our study will be entirely Lorentz covariant. Indeed this is certainly the simplest way to compute the necessary corrections from a nonrelativistic treatment. However, since such a "nonrelativistic" treatment of polarization seems still to occur more frequently in the published literature, at every step of our computation we shall explicitly exhibit the link between the two formalisms.

1. **Type of Required Experiment**

In the experiment, the \( \Sigma^0 \) must be polarized. Since it is produced by couplings assumed to preserve \( P \) and \( T \) invariance, the production reaction (on an unpolarized target at rest) must contain at least two linearly independent particle momenta. This excludes, for instance, \( \Sigma^0 \) production by \( K^- \) mesons stopped in hydrogen, but admits the collision

\[
K^- + p^+ \rightarrow \Sigma^0 + \pi^0. \tag{1}
\]

Other examples of possible reactions for the production of polarized \( \Sigma^0 \) are:

\[
\pi^- + p^+ \rightarrow \Sigma^0 + K^0, \tag{2}
\]

\[
\pi^- + p^+ \rightarrow \Sigma^0 + K^0, \tag{2}
\]

stopped \( K^- \):

\[
K^- + d^+ \rightarrow \Sigma^0 + p^+ + \pi^- \tag{3}
\]

or

\[
K^- + d^+ \rightarrow \Sigma^0 + n + \pi^0 \text{ (difficult to analyze).}
\]

Due to the large asymmetry in \( \Sigma^+ \rightarrow p^+ + \pi^0 \) decay,\(^4\) it is known that the \( \Sigma^0 \) produced in reactions similar to that of Eq. (3) is unpolarized,\(^6\) but those produced in the reaction \( \pi^- + p^+ \rightarrow \Sigma^+ + K^- \), which corresponds to Eq. (2) by charge independence, with a one-Gev \( \pi^+ \)-beam, have a degree of polarization\(^4\) \(|\eta_\pi| > 0.7 \pm 0.3\). We shall show in the Appendix that the present experimental data on cross sections for \( \Sigma^+ + \pi^- \) production\(^6\) in reactions similar to that of Eq. (2) imply a similar high degree of polarization for the \( \Sigma^0 \) produced in the reaction of Eq. (2) with a one-Gev \( \pi^- \) beam. This favors the choice of reaction (2) for the proposed experiment. On the other hand, we shall see in Sec. 9 that a measure of a lower bound of \(|\eta_\pi|\), the degree of polarization of \( \Sigma^0 \), will be a necessary by-product of the measurement of \( \varepsilon \).

In the decay of a polarized \( \Sigma^0 \) into \( \Lambda^0 + \gamma \) both final particles are polarized, but only the correlation between the photon transverse polarization and the \( \Lambda^0 \) polarization depends on \( \varepsilon \). The only possible way to measure such a correlation by present day experimental techniques is to observe in the same decay, the products of the \( \Lambda^0 \) disintegration and an electron pair produced by the photon. A schematic diagram of the corresponding bubble-chamber picture is drawn in Fig. 1(a) (for the case of reaction 2). The electron pair can be produced directly by \( \Sigma^0 \rightarrow \Lambda^0 + e^+ + e^- \); it is then called a Dalitz pair; the virtual photon producing it is quasi-real. Although the branching ratio

\[
\Sigma^0 \rightarrow \Lambda^0 + e^+ + e^-, \]

relative to

\[
\Sigma^0 \rightarrow \Lambda^0 + \gamma,
\]

is not large it has been computed\(^1\) and found to be


\(^7\)G. Feinberg, Phys. Rev. 109, 1019 (1958).
1/182 for $\epsilon=1$ and 1/161 for $\epsilon=-1$), the Dalitz pairs are somewhat more convenient for the measurement of the polarization correlation. So we establish the correlation function for both cases: ordinary electron pairs and Dalitz pairs. In the latter case the schematic diagram of the corresponding bubble-chamber picture is drawn in Fig. 1(b).

2. Method of Theoretical Analysis
(See Bernstein and Michel\textsuperscript{8} for a somewhat similar analysis.)

The polarization state of the $\Sigma^0$ is represented by a 2 by 2 density matrix $\rho_S$. The $S$ matrix for the decay is computed up to a factor. Then $R=\rho_S S^\dagger$ is the 4 by 4 density matrix which describes the polarization of the $\gamma-\Lambda^0$ system. The $\Lambda^0$ decay as a $\Lambda^0$-polarization analyzer, and pair production as analyzer for plane polarization of the photon, are represented by 2 by 2 Hermitian matrices, denoted, respectively, by $A_\lambda$ and $B_\gamma$. Then $F(\epsilon)=\text{Tr}R(B_\gamma\otimes A_\lambda)$ is the correlation function we want to compute, where $\otimes$ means the direct product of the two matrices.

More than eight particles are involved in the schemes of Fig. 1(a) or 1(b). In order to avoid for each physical quantity the use of an index indicating to which particle it belongs, we have to use many different letters. Table I is a complete summary of our notation.

3. Covariant Description of Spin $\frac{1}{2}$ Particle Polarization

For a given energy momentum $p$ we can choose two orthogonal states of polarization represented by the normed kets $\langle+\rangle$ and $\langle-\rangle$ denoted by $\lambda$ with $(\lambda,\mu)=\delta_{\lambda\mu}$. An arbitrary pure polarization state is represented by the normalized ket $\langle\xi\rangle=\xi\lambda$, where $(\xi,\xi)=|\xi|^2=1$. One can also represent it by the projector $\xi\xi^\dagger$, i.e., $\xi^\dagger=\frac{1}{2}(1+\xi\xi^\dagger)$, where $\sigma^I$ are the three Pauli matrices. $\xi\xi^\dagger$ is a shorthand for $\sum_{\lambda}\xi_{\lambda}\xi_{\lambda}^\dagger$. The normalization yields $\sum_{\lambda}|\xi_{\lambda}|^2=1$. The projector $\xi\xi^\dagger$ is called the density matrix of the state. If we do not consider pure states only but include partially polarized states, the density matrix for the polarization of the particle is still

$$\rho=\frac{1}{2}(1+\xi\xi^\dagger),$$

but then

$$0\leq|\xi\xi^\dagger|\leq1,$$

for $|\xi\xi^\dagger|$ is the degree of polarization. The set of three $\xi$, i.e., $\xi=\text{Tr}\rho\sigma$ is called the “Stokes vector.” For a particle at rest, $\sigma$ represents the spin operator (actually it is twice the infinitesimal rotation operator) and $\xi$, its mean value, is a genuine pseudovector in the three-dimensional space.

For a spin $\frac{1}{2}$ particle of energy-momentum $p$, it is

\footnote{J. Bernstein and L. Michel, Phys. Rev. 118, 871 (1960).}

\footnote{L. Michel and A. S. Wightman, Phys. Rev. 98, 1190 (1955).}

\footnote{C. Bouchiat and L. Michel, Nuclear Phys. 5, 416 (1958).}

\footnote{L. Michel, Suppl. Nuovo cimento 14, 95 (1959).}
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TABLE I. Complete summary of notations.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Beam</th>
<th>Σ⁰</th>
<th>Λ⁰</th>
<th>Decay product of Λ⁰</th>
<th>Ordinary pair</th>
<th>Dalitz pair</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>p₀</td>
<td>M'</td>
<td>M</td>
<td>μ⁺° μ⁻° μ° μ°</td>
<td>m⁺ m⁻ m⁺ m⁻</td>
<td>p⁺ p⁻ p⁺ p⁻</td>
</tr>
<tr>
<td>Momentum</td>
<td>p₁</td>
<td>p⁺</td>
<td>M'</td>
<td>μ⁺ μ⁻ μ° μ°</td>
<td>m⁺ m⁻ m⁺ m⁻</td>
<td>p⁺ p⁻ p⁺ p⁻</td>
</tr>
<tr>
<td>Energy-momentum four-vectors</td>
<td>p₂</td>
<td>p⁺</td>
<td>M'</td>
<td>μ⁺ μ⁻ μ° μ°</td>
<td>m⁺ m⁻ m⁺ m⁻</td>
<td>p⁺ p⁻ p⁺ p⁻</td>
</tr>
<tr>
<td>Covariantization</td>
<td>p₃</td>
<td>p⁺</td>
<td>M'</td>
<td>μ⁺ μ⁻ μ° μ°</td>
<td>m⁺ m⁻ m⁺ m⁻</td>
<td>p⁺ p⁻ p⁺ p⁻</td>
</tr>
<tr>
<td>Polarization Stokes vector</td>
<td>p₄</td>
<td>p⁺</td>
<td>M'</td>
<td>μ⁺ μ⁻ μ° μ°</td>
<td>m⁺ m⁻ m⁺ m⁻</td>
<td>p⁺ p⁻ p⁺ p⁻</td>
</tr>
</tbody>
</table>

Units time-like vectors used:

\[ t = (0, t') \]

Units space-like vectors used:

\[ \mathbf{n}(0), \mathbf{n}(0), \mathbf{n}(0), \mathbf{n}(0) \text{ [defined in Sec. 5, mainly Eq. (33)]}; \]

\[ b, \tau, \alpha = K m^n \] [see Eq. (43)];

\[ b \text{ defined in (50)}. \]

In lab. b = (0, \mathbf{b}) with \[ \mathbf{\delta} = (\mathbf{p}_0 \times \mathbf{p'})/(\mathbf{p}_0 \times \mathbf{p'}). \]

Numerical constants introduced:

\[ K_1 = (M^2 + M^2)/(M^2 + M^2) = 15.326; \]

\[ K_2 = 2M^2'(M^2 + M^2) = 15.294; \]

\[ K_3 = 1; \]

\[ K_4 = 2M^2(M^2 - M^2)/2M = 1.230; \]

where

\[ \Delta(M, m, p) = \sum (M + m + p) \sum (M - m - p) / (M - m - p). \]

Parameters \( \eta, \alpha, \beta \) satisfy \(-1 < \eta, \alpha, \beta < 1; \epsilon = \pm 1. \)

Orthogonality relation between four-vectors:

\[ \mathbf{n}(0), \mathbf{n}(0), \mathbf{n}(0), \mathbf{n}(0) \text{ [defined in Sec. 5, mainly Eq. (33)]}; \]

\[ \mathbf{b}, \mathbf{\tau}, \mathbf{\alpha} = K m^n \] [see Eq. (43)];

\[ \mathbf{b} \text{ defined in (50)}. \]

In lab. b = (0, \mathbf{b}) with \[ \mathbf{\delta} = (\mathbf{p}_0 \times \mathbf{p'})/(\mathbf{p}_0 \times \mathbf{p'}). \]

Relation between vectors:

\[ \mathbf{n}(0) = K_{11} \mathbf{u}' + K_{22} \mathbf{u}; \]

\[ \mathbf{n}(0) = -K_{11} \mathbf{u} + K_{22} \mathbf{u}. \]

Conservation of energy and momentum.

In \( \Sigma^0 \rightarrow \Lambda^0 + \gamma \):

\[ M^0 = M^0 + E \]

\[ \mathbf{n}(0) = M^0 + \mathbf{f} \]

\[ \mathbf{u} : \mathbf{u}' = K_{11} / K_{22}. \]

In \( \Lambda^0 \rightarrow \rho^0 + \pi^0 \):

\[ M^0 = m^0 + m^0 \]

\[ \mathbf{n}(0) = \mathbf{u} : \mathbf{u}' = K_{44}. \]

Furthermore, for that decay

\[ \mathbf{n}(0) = K_{11} \mathbf{u}' + K_{22} \mathbf{u}; \]

\[ \mathbf{n}(0) = -K_{11} \mathbf{u} + K_{22} \mathbf{u}. \]

with \( \mathbf{\epsilon} \cdot \mathbf{n} = 1. \) The completeness relation yields

\[ g_{\alpha \beta} \mathbf{n}(0) \mathbf{n}(\rho) = g_{\alpha \beta}. \]

For the particle with energy momentum \( \mathbf{p} \) and mass \( m \),

\[ \mathbf{n}(0) \text{ with } i = 1, 2, 3, \text{ be such a right-handed orthonormal base that we shall call shortly a "tetrad."} \]

\[ \mathbf{\xi} \text{ are the components of } \mathbf{\xi} \text{ in this tetrad: } \]

\[ \mathbf{\xi} = \sum \mathbf{\xi} \mathbf{n}(i); \]

\[ \mathbf{\xi} = \mathbf{\xi} \mathbf{n}; \]

\[ \mathbf{\xi} = -\mathbf{n} \cdot \xi \mathbf{n}. \]

We obtain for the square of the degree of polarization, the equivalent expressions:

\[ \mathbf{n} = \xi \mathbf{n} \text{ and } \mathbf{\xi} = -\mathbf{n} \cdot \xi \mathbf{n}. \]

We denote by \( \mathbf{\xi}', \mathbf{\eta}' \) a tetrad associated with the \( \Sigma^0 \).

\[ \rho = \frac{1}{2}(1 - \mathbf{\xi} \cdot \mathbf{\eta}) = \frac{1}{2}(1 - \mathbf{\xi} \cdot \mathbf{\eta}) \]

\[ \rho = \frac{1}{2}(1 + \mathbf{\xi} \cdot \mathbf{\eta}) = \frac{1}{2}(1 + \mathbf{\xi} \cdot \mathbf{\eta}) \]

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4. Covariant Description of Photon Polarization

Since a photon of given momentum has two linearly independent states of polarization, its 2 by 2 density matrix for polarization is similar to that of spin \( \frac{1}{2} \) particles. Let us choose for basic states \( \lambda \), with \( \lambda = \pm \), the right and left circular polarization. An arbitrary pure state of polarization is \( \xi = \xi \lambda \) and the density matrix is \( \xi \xi = \frac{1}{2} (1 + \gamma \cdot \sigma) \) with \( \gamma = (\xi \sigma \xi) \) and \( \gamma^2 = 1 \). Since \( \gamma_3 \) is diagonal, \( \gamma_3 \) = \(+1\) corresponds to pure right circular polarization, \( \gamma_3 = -1 \) to pure left circular polarization, and \( \gamma_3 = 0 \) to plane polarization.

For partial polarization, the density matrix has the same form:

\[
\rho = \frac{1}{2} (1 + \gamma \cdot \sigma),
\]

where

\[
0 \leq |\gamma| = (\gamma^2)^{1/2} = \text{degree of polarization} \leq 1.
\]

Although the three numbers \( \gamma_1 \) can never be the components of a vector, their set \( \gamma \) is often called the "Stokes vector" since (three linear combinations of) the \( \gamma_i \) were introduced by Stokes\(^{16}\) in 1852. The use of \( \gamma_i \) is well spread nowadays.\(^{13,14}\)

We recall here how the Stokes vector is related to the covariant formalism:\(^{13,14}\) since the situation is now radically different from that of spin \( \frac{1}{2} \) particles.

One shows that \( \gamma_3 \) is a pseudoscalar and \( \gamma_\sigma = (\gamma_1^2 + \gamma_2^2)^{1/2} \) is a scalar for the Lorentz group, so instead, we use the vocabulary of elliptical polarization. We call \( \gamma = |\gamma_3| \) and \( \gamma_\sigma \) the degrees of circular and plane polarization.

Since \( \psi = 0 \) there are only two other linearly independent four-vectors orthogonal to \( \psi \); we denote them by \( \psi^{(0)} \) and \( \psi^{(2)} \); they satisfy

\[
i = 1, 2: \quad \psi = \psi \cdot \psi^{(0)} = 0, \quad -\psi^{(0)} \cdot \psi^{(0)} = \delta_{ij}, \quad k \cdot \psi^{(0)} \times \psi^{(2)} > 0.
\]

Note that the \( \psi^{(0)} \) are defined up to an arbitrary component along \( \psi \). The photon polarization vector

\[
e = \xi \psi^{(0)} + \xi \psi^{(2)},
\]

which describes pure states of polarization, is a genuine vector orthogonal to \( \psi \) and defined up to a component along \( \psi \) [in the choice \( e = (0, e) \), \( e \) is proportional to the photon electric vector] but its length is defined in an Hermitian metric:

\[
(e, e) = 1 = |\xi_1|^2 + |\xi_2|^2.
\]

In a real base, such as (19), we can define the complex conjugated vector: \( e^* = \xi^* \psi^{(0)} + \xi^* \psi^{(2)} \), and we have the following identity for vectors orthogonal to \( \psi \):

\[
(a, b) = -a^* \cdot b.
\]

The circular polarization vector

\[
\epsilon_s = \frac{1}{\sqrt{2}} (\psi^{(0)} \pm i \psi^{(2)}),
\]

satisfies

\[
(\epsilon_s, \epsilon_s) = \delta_\sigma = -\epsilon_s^* \cdot \epsilon_s \quad \text{and} \quad \epsilon_s^* = \epsilon_s.
\]

Then, the polarization vector can be expanded:

\[
e = \xi \epsilon_s
\]

where

\[
\xi_\lambda = (\epsilon_s, e) = \frac{1}{\sqrt{2}} (\xi_1 - i \xi_2) = -\epsilon_s^* \cdot e.
\]

The \( \xi_\lambda \) are also the component of the representative ket introduced in the beginning of this section. Using this isomorphism between \( \xi \) and \( e \), we can construct the density matrix in terms of a tensor orthogonal to \( \psi \). For a pure state,

\[
\rho = -e \otimes e^* = -\frac{1}{2} (1 + \gamma \cdot \sigma) \epsilon_s \otimes \epsilon_s^*.
\]

The right-hand side represents also partial polarization when \( 0 \leq |\gamma| \leq 1 \).

Indeed, in this isomorphism the unit matrix represents the tensor

\[
I = -\sum_\lambda \delta_\lambda \epsilon_s \otimes \epsilon_s^* = -\sum_{ij} \delta_{ij} \psi^{(0)} \otimes \psi^{(0)},
\]

and the Pauli matrices represent the tensors

\[
P_k = -\sum_\lambda (\gamma_\sigma)_\lambda \epsilon_s \otimes \epsilon_s^*,
\]

\[
P_k' = -\sum_{ij} (\gamma_\sigma)^{ij} \psi^{(0)} \otimes \psi^{(0)},
\]

depending on which basis \( (\epsilon_s, \psi^{(0)}) \) is chosen for polarization vectors. We leave it to the reader to prove that

\[
P_1 = P_1', \quad P_2 = P_2', \quad P_3 = P_3',
\]

and that a photon density matrix can always be written covariantly as

\[
\rho = \frac{1}{2} [I (1 - \gamma) - 2 \gamma e \otimes e^*],
\]

where \( \gamma \) is the degree of polarization and \( e \) a unit complex vector (i.e., \(-e \cdot e^* = 1\)) orthogonal to \( \psi \) (it is defined up to a component along \( \psi \)).

5. The (up to a Factor) \( S \) Matrix for \( \Sigma^0 \) Decay

To express \( S \) explicitly in a chosen basis, we have first to choose the tetrads associated with \( \Sigma^0 \) and \( \Lambda^0 \). Let us consider the \( \Sigma^0 \rightarrow \Lambda^0 + \gamma \) decay with given energy-momenta which satisfy

\[
\psi' = M' \psi + \psi = Mu + f.
\]

We shall choose \( \psi^{(o)} \) = \( \psi^{(0)} \) and \( \psi^{(2)} = \psi^{(2)} \) in the 2-plane (i.e., the two-dimensional plane) orthogonal to \( \psi', \psi, \psi \) and such that they satisfy (19). Then \( \psi' \) and \( \psi_2 \) are

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\(^{16}\) G. G. Stokes, Proc. Cambridge Phil. Soc. 9, 399 (1852).
7. The $\Lambda^0$ Decay as $\Lambda^0$-Polarization Analyzer

An observable is described by an Hermitian operator. For a polarization analyzer the part of the Hermitian operator corresponding to the polarization of a spin $\frac{1}{2}$ matrix with well-defined momentum is a 2 by 2 Hermitian matrix which can be written

$$ A = \chi (1 + \alpha \cdot s), $$

and the probability of “counting” the particle whose polarization matrix is $\rho$ [written in (4)] is

$$ \text{Tr} \rho A = \chi (1 + \alpha \cdot \xi). $$

This shows that $\chi$ is the probability (or counting rate or cross section, etc.) for the observation of unpolarized particles and that $0 \leq \alpha^2 \leq 1$; indeed $|\alpha|$ is the efficiency of the polarization analyzer and the direction of $\alpha$ corresponds to the “setting” of this apparatus. In the following we shall use a unit “Stokes vector” $\lambda$ for the description of the setting of the apparatus; then

$$ \alpha = a \lambda $$

so the covariant form of the matrix $A$ defined in Eq. (39) is

$$ A = (1 - a \alpha \cdot \xi), $$

and, with $\rho$ given in (15):

$$ \text{Tr} \rho A = (1 - a \alpha \cdot \delta). $$

Let us consider a $\Lambda^0$ with energy momentum $p = M \mathbf{u}$ and polarization $\mathbf{\xi}$ decaying into a proton and a $\pi^-$ meson with energy-momenta $q = m \mathbf{v}$ and $q' = m' \mathbf{v}'$. Spin $\frac{1}{2}$ for the $\Lambda^0$ implies that the transition probability is linear in $\mathbf{v}$ and nonconservation of parity implies that it is the sum of a scalar and of a pseudoscalar; its most general form is therefore $1 - a K \mathbf{v}' \cdot \delta$. The choice of $\mathbf{v}'$ and not of $\mathbf{v}$ is in agreement with general use. Only $\mathbf{v}' = (\mathbf{v} - (\mathbf{v} \cdot \mathbf{u}) \mathbf{u})$, the component of $\mathbf{v}'$ orthogonal to $\mathbf{u}$, is significant. The constant $K_3$ is such that $K_3 \mathbf{v}'$ is a unit vector and so $-1 \leq a \leq 1$. The value of $K_3$ is given in Table I.

The comparison of the $\Lambda^0$ decay rate with (44) shows that the matrix $A_\Lambda$ representing $\Lambda^0$-decay as $\Lambda^0$-polarization analyzer is proportional to

$$ A = 1 - a K_3 \mathbf{v}' \cdot \mathbf{u} \cdot \mathbf{\xi} = 1 - a K_3 \mathbf{v}' \cdot \mathbf{u} \cdot \mathbf{\xi}. $$

Let $\eta_\perp$ be the transverse polarization of the $\Lambda$-hyperon particle produced in a given reaction ($\eta_\perp > 0$ if the polarization $\xi$ is along the direction $+$ of $\mathbf{p}_3 \times \mathbf{p}_4$, and $\eta_\perp < 0$ if $\xi$ is along the direction $-$ of $\mathbf{p}_3 \times \mathbf{p}_4$). Some asymmetry measurements for $\Lambda^0$ decay have yielded $\eta_\perp A = 0.73 \pm 0.14$; this value is a lower limit on $|\alpha|$.
(and most physicists believe that \( |\alpha| = 1 \). As we shall see, we do not need the sign of \( \alpha \), but the product \( \alpha \eta \) (we recall that \( \eta \) is the sign and degree of \( \Sigma^0 \) polarization) will have to be measured in the same experiment.

8. Pair Production as \( \gamma \)-Plane Polarization Analyzer

As Stokes pointed out more than a century ago, a light polarization analyzer is to be described by a 2 by 2 Hermitian matrix. We can write it as in (39): \( B = \chi^2 (1 + \beta \gamma) + \chi \), and then, with (17), we find for the “counting” rate:

\[
\text{Tr} B_{\gamma} = \chi (1 + \beta \gamma) ;
\]

or we can write \( B \) as in Eq. (31), namely:

\[
B = \chi \left[ I (1 - \beta) - 2 \beta \mathbf{b} \otimes \mathbf{b}^* \right],
\]

where \( \beta \) is the efficiency of the process as polarization analyzer and the unit vector \( \mathbf{b} \) is the “setting.” The transition rate is then

\[
\text{Tr} B_{\gamma} = \chi (1 - \beta \gamma + 2 \beta \gamma |\mathbf{b} \cdot \mathbf{e}^*|). \tag{48}
\]

If \( \mathbf{b} \) and/or \( \mathbf{e} \) are real (as it is in the case of plane polarization) this can be written:

\[
\text{Tr} B_{\gamma} = (1 + \beta \gamma \cos 2\phi), \tag{49}
\]

where \( \cos \phi = -\mathbf{e} \cdot \mathbf{b} \).

In the proposed experiment we are not interested in the photon circular polarization since in (38) the coefficients of \( \tau_3 \) matrices, which correspond to \( \beta \)-circular polarization, do not contain \( \epsilon \). Note, however, that if this circular polarization measurement can be performed, it would give the value of both \( \alpha \) and \( \eta \) separately (including their sign). This would be a very important result. However, this experiment cannot be performed, with present experimental techniques, in a bubble chamber. For instance, pair production is a poor analyzer of the circular polarization of high-energy photons (efficiency \( \beta \) of the order of \( m/e^2 \)). On the other hand, Dalitz pairs do not analyze circular polarization.

The most efficient phenomenon for analyzing high-energy photon plane polarization seems to be electron pair production. (Compton scattering has a too low \( \beta \), nuclear photoeffects a too low \( \chi \).) The corresponding \( \beta \) and \( \mathbf{b} \) are complicated functions of \( I \), \( \mathbf{p}^+ \), \( \mathbf{p}^- \). We shall not give them explicitly. However, the angles between \( \mathbf{k}, \mathbf{p}^+, \mathbf{p}^- \) are difficult to measure (they are small and there is multiple scattering). If the only measured angle is \( \phi_\alpha \), the azimuth around \( \mathbf{k} \) of the normal \( \mathbf{p}_+ \times \mathbf{p}_- \) of the plane of the pair, then \( \mathbf{b} = (0, \mathbf{b}) \), with \( \mathbf{b} \) the unit vector of

\[
(\mathbf{k} \cdot \mathbf{p}_+)(\mathbf{k} \times \mathbf{p}_+) - (\mathbf{k} \cdot \mathbf{p}_-)(\mathbf{k} \times \mathbf{p}_-), \tag{50}
\]

and the corresponding \( \beta \) has been computed by Karlson.\(^{16}\) Figure 2 gives \( \beta \) for 66-Mev photons, as a function of \( (E_+ - E_-)/(E_+ + E_-) \), the repartition of energy in the pair.\(^{16}\)

9. Correlation Functions for \( \Sigma^0 \to \Lambda^0 + \gamma \) Decay

We have to define a notation for “partial traces.” Consider a 4 by 4 matrix \( Z = \sum \epsilon_i \Gamma_i \otimes \Lambda_i \), where \( \Gamma_i \) and \( \Lambda_i \) are 2 by 2 matrices. We define

\[
\text{Tr}_{\gamma} Z = \sum \epsilon_i \langle \text{Tr} \Gamma_i \rangle \Lambda_i, \quad \text{Tr}_{\gamma} \Lambda = \sum \epsilon_i \langle \text{Tr} \Gamma_i \rangle \langle \text{Tr} \Lambda_i \rangle \tag{51}
\]

We verify that

\[
\text{Tr} Z = \text{Tr}_{\gamma} \langle \text{Tr} \Lambda \rangle \text{Tr}_{\gamma} Z = \sum \epsilon_i \langle \text{Tr} \Gamma_i \rangle \langle \text{Tr} \Lambda_i \rangle \text{Tr}_{\gamma} \Lambda \tag{52}
\]

The matrix \( R \), Eq. (37) or (38), contains all possible information concerning the particle polarizations. For instance, if we observe the \( \Lambda^0 \) by the analyzer represented by \( A_\Lambda \), the photon is in the state \( \text{Tr}_{\gamma} R (1 \otimes A_\Lambda) \).

Conversely, let us suppose that we do not observe the \( \gamma \)-polarization \( (B_\gamma = 1) \). Then the \( \Lambda^0 \) polarization is described by

\[
\text{Tr}_{\gamma} R = \frac{1}{2} (1 - \eta \delta_{\tau_3}) = \frac{1}{2} (1 + \eta \beta \cdot \mathbf{e} \cdot \mathbf{b} \tau_3), \tag{53}
\]

and the correlation function yielded by \( \Lambda^0 \) decay only (nonobservation of the \( \gamma \)) is then

\[
H = \text{Tr} R (1 \otimes A_\Lambda) = \text{Tr} (A_\Lambda \text{Tr}_{\gamma} R),
\]

\[
H = 1 + \alpha \eta K_2 K_3 (\mathbf{u} \cdot \mathbf{b}) \mathbf{u} \cdot \mathbf{b} \cdot \mathbf{u}, \tag{54}
\]

This correlation function shows how \( \alpha \eta \) can be measured. While \( \alpha \) is a universal constant characterizing \( \Lambda^0 \) decay, \( \eta \) is expected to be a function of the beam energy and the angle of production of \( \Sigma^0 \) or, in terms of four-vectors, a function of \( \mathbf{p}_\gamma \) and \( \mathbf{u}_\gamma \). (See also the Appendix.)

\(^{16}\) The sign of \( \beta \) has been the subject of some controversy; see, e.g., T. H. Berlin and L. Madansky, Phys. Rev. 78, 623 (1950), G. C. Wick, Phys. Rev. 81, 467 (1951), and reference 17. It is true, indeed, that the sign of \( \beta \) is opposite for ordinary pairs \( (\beta < 0) \) and for Dalitz pairs \( (\beta > 0) \) as we shall see in Sec. 10.
Instead, to compute directly the final correlation function \( F(\epsilon) \), let us proceed by steps. If the \( \Lambda^0 \) decay is observed, what is the polarization state of the \( \gamma \)?

\[
\rho_\gamma = \mathbf{R}(10 \gamma \Lambda) = \frac{1}{2} \{ 1 - 2 \alpha \lambda \delta + (\eta \delta - \alpha \lambda) \tau_2 - \epsilon \alpha \epsilon' \} = \frac{1}{2} \{ 1 - 2 \alpha \lambda \delta + (\eta \delta - \alpha \lambda) \tau_2 \}.
\]

(55)

The terms \( (\eta \delta - \alpha \lambda) \tau_2 \) are for partial circular polarization. In this section we are interested only in the plane polarization and we shall drop these \( \tau_2 \) terms.

Then Eq. (55) can be written, with \( I \) defined in Eq. (28) and \( H \) in Eq. (54):

\[
\rho_\gamma = \frac{1}{2} \{ H + \epsilon \alpha \epsilon \}.
\]

(56)

where

\[
\mathbf{b} \cdot \mathbf{a} = \mathbf{b} \cdot \mathbf{a} + (\mathbf{b} \cdot \mathbf{a}')(\mathbf{a} \cdot \mathbf{a}').
\]

\[
= K_\alpha \mathbf{b} \cdot \mathbf{w'} - K_\alpha \mathbf{b} \cdot \mathbf{w} - K_\alpha K_\alpha.
\]

The use of Eq. (46) for \( B \) gives us the correlation function \( F \) as

\[
F(\epsilon) = 1 - \alpha \epsilon [K_\alpha G + \frac{1 + \epsilon}{2} (\mathbf{b} \cdot \mathbf{w'} + K_\alpha G)]
\]

(57)

\[
+ \epsilon \mathbf{b} \cdot \mathbf{b} \cdot \mathbf{w'},
\]

\[
F'(\epsilon) = 1 - \alpha \epsilon [K_\alpha G + \frac{(\mathbf{b} \cdot \mathbf{w'}) + K_\alpha G}{2} (D + r^2 - r'^2)]
\]

(61)

\[
F(\epsilon) = 1 - \alpha \epsilon [K_\alpha G + \frac{r^2 - r'^2}{2} (\mathbf{b} \cdot \mathbf{w}')(\mathbf{b} \cdot \mathbf{w}')] + \epsilon \mathbf{b} \cdot \mathbf{b} \cdot \mathbf{w'},
\]

(62)

where \( G \) is given in (58).

We can also write \( F(\epsilon) \) in a form similar to (49), which shows better its structure and also its relation with noncovariant formalism:

\[
F(\epsilon) = 1 - \alpha \epsilon [\cos \beta \cos \phi_1 + \epsilon \sin \phi_1 \sin \phi_2 \cos \phi],
\]

(64)

where

\[
2 \phi = 2 \phi_2 - \phi_1 - \phi_2,
\]

(65)

and the quantities \( \phi_1, \phi_2, \phi_3, \phi_4, \) and \( \phi_5 \) are defined as follows:

\[
- \mathbf{b} \cdot \mathbf{n}^{(\prime)} = \delta_2 = \cos \phi_2,
\]

\[
- \mathbf{b} \cdot \mathbf{n}^{(\prime)} = \delta_1 = \sin \phi_1 \cos \phi_2,
\]

\[
- \mathbf{b} \cdot \mathbf{n}^{(\prime)2} = \sin \phi_2 \sin \phi_3,
\]

\[
- K_\alpha \mathbf{w'} \cdot \mathbf{n}^{(\prime)} = \lambda_2 = \cos \phi_2,
\]

\[
- K_\alpha \mathbf{w'} \cdot \mathbf{n}^{(\prime)2} = \lambda_1 = \sin \phi_1 \cos \phi_2,
\]

\[
- K_\alpha \mathbf{w'} \cdot \mathbf{n}^{(\prime)2} = \sin \phi_2 \sin \phi_3,
\]

\[
- \mathbf{b} \cdot \mathbf{n}^{(\prime)} = \cos \phi_3,
\]

\[
- \mathbf{b} \cdot \mathbf{n}^{(\prime)2} = \sin \phi_3.
\]

(66)

Except for an arbitrary and immaterial parameter (origin of the azimuth around \( k \)) for \( \mathbf{n}^{(\prime)} \), the vectors \( \mathbf{n}^{(\prime)}, \mathbf{n}^{(\prime)2}, \mathbf{n}^{(\prime)2} \) have been defined in paragraph 5.

The usual "nonrelativistic" treatment proceeds as follows: Let us suppose the photon with a plane polarization \( \epsilon \) corresponding to the azimuth \( \phi \) around \( k \),

\[
G = \mathbf{b} \cdot (K_\lambda K_\lambda - K_\lambda' K_\lambda'),
\]

and \( \mathbf{b} \) is defined in (50) and \( \beta \) drawn in Fig. 2.

For the sake of completeness we also give \( I \) as a function of \( p_4 \) and \( p_5 \), although it has no practical value for the discussed experiment. In the literature, the cross section for electron pair production by a totally plane polarized photon is given by (see May,\textsuperscript{17} also reference 14, p. 374) (up to a factor)

\[
D = - (r \cdot \epsilon)(r' \cdot \epsilon),
\]

(59)

where \( t = (1, 0) \),

\[
D = \frac{|k \times (p_5 + p_-)|^2}{(f \cdot p_+)(f \cdot p_-)} = \frac{t \cdot p_+}{f \cdot p_-} - \frac{t \cdot p_-}{f \cdot p_+} - (f \cdot t),
\]

(60)

\[
\tau' = |k - p_+ - p_-| = \frac{1}{f \cdot p_+} - \frac{1}{f \cdot p_-}.
\]

The corresponding 2 by 2 matrix is

\[
B' = D + r^2 - r'^2,
\]

(61)

\[
\text{and the corresponding correlation function } F' \text{ is}
\]

\[
F'(\epsilon) = 1 - \alpha \epsilon [K_\alpha G + \frac{r^2 - r'^2}{2} (\mathbf{b} \cdot \mathbf{w'})(\mathbf{b} \cdot \mathbf{w'}) + (\mathbf{b} \cdot \mathbf{w})(\mathbf{b} \cdot \mathbf{w})]
\]

(63)

\[
i.e. \text{ [see (26)]}
\]

\[
\epsilon = (\mathbf{n} \cdot \epsilon \cos \phi + \mathbf{n} \cdot \epsilon \sin \phi) = \frac{1}{\sqrt{2}} (e^{i\epsilon} \epsilon_+ + e^{i\epsilon} \epsilon_-)
\]

(67)

where \( \gamma = \pm 1 \).

The \( S \) matrix between \( \Sigma^0 \) and \( \Lambda^0 \) polarization states is then [see (34)] \( S' \) such that

\[
\lambda'=S'_{\lambda \sigma}\lambda,
\]

(68)

\[
\text{with}
\]

\[
S'_{\lambda \sigma} = \frac{1}{\sqrt{2}} \sum \epsilon e^{-i\gamma \phi} \gamma S_{\lambda \sigma},
\]

(69)

or

\[
\sqrt{2} S' = \begin{pmatrix} 0 & -e^{-i\phi} \\ e^{i\phi} & 0 \end{pmatrix}
\]

\[
= \tau_1 \cos \phi + \tau_2 \sin \phi \text{ when } \epsilon = -1
\]

\[
= -i \left[ \tau_1 \cos \left( \phi + \frac{\pi}{2} \right) + \tau_2 \sin \left( \phi + \frac{\pi}{2} \right) \right]
\]

(70)

\[
\text{when } \epsilon = 1.
\]

\textsuperscript{17} M. M. May, Phys. Rev. 84, 265 (1951).
This can be written, up to a factor,  

\[ S' = (e \cdot e'), \quad \text{when} \quad e = -1, \quad e' = e \]  

when \( e = 1, \quad e' = k \times e / |k| \). \hspace{1cm} (72)

Generally, physicists postulate directly these forms of \( S' \) on parity conservation and “rotational invariance” grounds. It is very clear how this has to be interpreted. Using \( \rho_z \) in (16), we can compute  

\[ \rho_z = S'_{z2} S'^{11} = \frac{1}{2} S'(1 + y \delta \cdot \tau) S'^{11} = \frac{1}{2} (1 + \xi \cdot \tau), \quad (73) \]

where  

\[ \xi = -\eta \bar{d} - 2e'(e' \cdot \bar{d}). \]

In plain words, the \( \Lambda^0 \)-polarization is obtained from \( \Sigma^0 \)-polarization by a rotation of \( \pi \) around \( e' \). We obtain easily \( F(e) \) as in (64) if we take \( T_{z2} R_{12} \), with \( \gamma_1 = \beta \cos \phi \), \( \gamma_2 = \beta \sin \phi \), \( \gamma_3 = 0 \). We have also explicitly displayed the relativistic meaning of the three-component Stokes vectors used in the so-called “non-relativistic” formalism.

10. Correlation Functions for \( \Sigma^0 \to \Lambda^0 + e^+ + e^- \) Decay

Although they are rare, decays with a Dalitz pair are much more interesting from the experimental point of view. Indeed the angle between \( p'_e \) and \( p'_{\mu} \) is large, on the average, than that for ordinary pairs, so the direction of the normal to the plane of the pair can be determined for Dalitz pairs while multiple scattering make this barely possible for ordinary pairs in a hydrogen bubble chamber. Also, as we shall see, the efficiency \( \beta \) of the Dalitz pair as a plane polarization analyzer is greater.

A refined theoretical treatment of the \( \Sigma^0 \to \Lambda^0 + e^+ + e^- \) decay\(^{18} \) would require the determination of the two independent form factors (for each value of \( e \)) of the \( \Sigma^0 \to \Lambda^0 \) current. However, as is seen from Kroll and Wada's\(^9 \) study of the general problem of Dalitz pairs, the azimuthal distribution of the plane of the pair is not sensitive to the detailed structure of the form factors.

In this section we shall give the value of \( \beta \) as a function of \( p'_e \) and \( p'_{\mu} \). For this, we define (as in reference 19)  

\[ \Sigma_{m_e} = p'_e + p'_{\mu}, \]

so that \( x^2 = y^2 \) is the square of the “virtual photon” mass (in electron mass units)  

\[ y = (E'_e - E'_{\mu}) / |p'_e + p'_{\mu}|, \hspace{1cm} (76) \]

[in practice \( y \) is the energy partition \( (E'_e - E'_{\mu}) / (E'_e + E'_{\mu}) \) of the pair].

Except for a slight modification of the constants \( K_i \)

\[^{18} \text{This is being done by one of us (H.R.). This paper is part of a work submitted as his thesis to the University of Paris.} \]

\[^{9} \text{N. Kroll and W. Wada, Phys. Rev. 98, 1355 (1955).} \]
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APPENDIX

Polarization of the $\Sigma^0$ Produced in the Reaction $\pi^- + p^+ \rightarrow \Sigma^0 + K^0$

If the proton is unpolarized, one can conclude from $P$ and $T$ invariance that the $\Sigma$ polarization is orthogonal to the plane of the reaction. We have represented the polarization by the pseudovector $\eta\delta$, where $\delta = (p_\pi \times p_p) \cdot p_K$ and $-1 \leq \delta \leq 1$. The parameter $\eta$ represents the sign and the degree of the polarization. Both are unknown. We shall first study in this Appendix what can be deduced for the value of $\eta$ from the present experimental data on the reactions:

$$\begin{align*}
\pi^- + p^+ &\rightarrow \Sigma^+ + K^+ , \\
\pi^- + p^+ &\rightarrow \Sigma^0 + K^0 , \\
\pi^- + p^+ &\rightarrow \Sigma^- + K^+ ,
\end{align*}$$

and from the hypothesis of charge independence.

The argument will be based on the following lemma about triangular relations. Let $a$, $b$, and $c$ be three positive numbers. The following relations are equivalent:

I. $a < b + c$, $b < a + c$, $c < a + b$,

II. $(a - b - c)(a - b + c)(a + b + c) \leq 0$,

or

II'. $\Delta(a,b,c) = (a + b + c)(a - b - c)(a - b + c)(a + b - c) \leq 0$,

III. $-2ab \leq a^2 + b^2 - c^2 \leq 2ab$.

Note that these relations also imply:

$$|a - b| \leq c , \quad |a - c| \leq b , \quad |b - c| \leq a .$$

These relations are called triangular relations. We note that $\Delta(a,b,c) \leq 0$ when $a$, $b$, and $c$ verify a triangular relation.

**Lemma 1.** If $\Delta(a_i,b_i,c_i) \leq 0$ for $n$ sets $a_i$, $b_i$, $c_i$, of positive numbers, then $\Delta(\sum_i a_i^2, b_i^2, c_i^2) \leq 0$.

For the proof, we let write relation III for each $i$ and add them up; we obtain

$$-2\sum_i a_i b_i \leq \sum_i (a_i^2 + b_i^2 - c_i^2) \leq 2\sum_i a_i b_i ,$$

(A.4)
By transitivity, (A.4) and (A.5) yield
\[ -2(\sum_i a_i^2)(\sum_j b_j^2) \leq \sum_i a_i^2 + \sum_j b_j^2 \]
\[ -\sum_i c_i^2 \leq 2(\sum_i a_i^2)(\sum_j b_j^2), \]
which proves the lemma.

**Lemma 2.** If
\[ \Delta(a_i,b_i,c_i) \leq 0 \quad \text{and} \quad \Delta(\sum_i a_i^2)(\sum_j b_j^2) = (\sum_i a_i^2)(\sum_j b_j^2), \]
then
\[ c_i = a_i + b_i \quad \text{and} \quad a_i/b_i = b_i/c_i. \]

Proof: By squaring (A.7) we obtain the first equality in (A.6) and from (A.4), (A.5), and (A.6) we obtain
\[ -2(\sum_i a_i^2)(\sum_j b_j^2) = -2\sum_i a_i b_i \]
\[ = \sum_i(a_i^2 + b_i^2 - c_i^2). \]
The first equality of (A.8) is equivalent to
\[ \sum_{i<j} (a_i b_j - a_j b_i)^2 = 0 \quad \text{or} \quad \frac{a_i}{b_i} = \frac{a_j}{b_j}. \]
The second equality of (A.8) is equivalent to \( \sum_i c_i^2 = \sum_i(a_i + b_i)^2 \) which, combined with relation I for each value of \( i \), yields \( c_i = a_i + b_i \). This with (A.9) proves the lemma.

If one assumes charge independence, the amplitude for three reactions (A.1), (A.2), (A.3) for given states of energy-momenta and polarizations satisfy the linear relation
\[ \sum_i a_i^2 = \sum_i a_i + b_i^2 \]
and hence a triangular relation
\[ \Delta(2a_0, \{a_i\}, \{a_\ldots\}) < 0, \]
The \( \Sigma \) polarizations \( \eta_+ \), \( \eta_0 \), \( \eta_- \) are functions of the same variables. The quantities \( a_\alpha(1 \pm \eta_\alpha) \) (where \( \alpha = +, 0, - \)) are the cross sections for production of totally polarized \( \Sigma \) (with polarization \( \pm \alpha \)); they satisfy
\[ \Delta(2a_0, \{a_\alpha\}, \{a_\ldots\}) < 0. \]
The \( \Sigma \) polarizations \( \eta_+ \), \( \eta_0 \), \( \eta_- \) are functions of the same variables. The quantities \( a_\alpha(1 \pm \eta_\alpha) \) (where \( \alpha = +, 0, - \)) are the cross sections for production of totally polarized \( \Sigma \) (with polarization \( \pm \alpha \)); they satisfy
\[ \Delta(2a_0, \{a_\alpha\}, \{a_\ldots\}) < 0. \]

The quantities already measured (for a beam energy around 1.1 Bev) are \( a_+ \), \( a_0 \), \( a_- \) and a lower limit for \( |\eta_+| \), i.e., \( |\eta_+| \geq 0.7 \pm 0.3 \) (see references 4, 5, 6). In these experimental data the relation
\[ (2a_0)^2 \leq (a_+)^2 + (a_-)^2 \]
is barely satisfied and a simplifying hypothesis, suggested in reference 6, is that for all angles (or at least for a large range of \( \theta \), the angle of production in the center-of-mass system), the cross sections satisfy the equality in (A.12). Then Lemma 2 tells us that
\[ \eta_+ = \eta_0 = \eta_- \quad \text{(A.13)} \]

We will now outline the principle of the easiest method for obtaining some information on \( |\eta_+| \). Indeed the \( K^0 \) meson produced in Eq. (A.2) is expected to be visible in only one third of the bubble-chamber pictures (one cannot see \( \theta_0 \) or \( \theta_0 \rightarrow 2 \pi^0 \)). Hence in most pictures one can measure only the energy-momentum \( p_\text{b} \) of the beam, that of the target: \( m_\text{t} = (m_0,0) \), and of the \( \Lambda^0; M \text{u} \). We define a unit time vector \( u'' \) and a “mass” \( M'' \) by
\[ p_\text{b} + Mt = M''u''. \]
Energy-momentum conservation in Eq. (A.2) gives
\[ M''u'' = M'u' + \mu Ku_K, \]
where \( \mu_K \) is the \( K^0 \) meson mass. We deduce
\[ u'' \cdot u' = (M''^2 + M'^2 - \mu^2K^2)/(2M'M'') = (1 + K^2)^2, \]
where
\[ K = (M'' - M' + \mu K)/(2M'M''). \]
Let \( u'' \), \( I, I' \) be the tetrad defined by
\[ I = (0,1) \quad \text{with} \quad I = \langle \mathbf{p}_0 \times \mathbf{u}', \mathbf{p}_0 \times \mathbf{u} \rangle, \]
and
\[ I' = (0,1) \quad \text{with} \quad I' = \langle \mathbf{p}_0 \times \mathbf{I}, \mathbf{p}_0 \times \mathbf{I} \rangle, \]
and let \( \theta \) be the angle of production in the rest-system of \( \Sigma \), and \( \varphi \) the azimuth of \( \mathbf{p}' \) around \( \mathbf{p}_0 \); i.e.,
\[ u'' = (1 + K^2)u'' + K \sin \theta \sin \varphi \]
\[ + K \cos \theta \sin \varphi I' + K \cos \theta I'' \]
and
\[ b = -I \sin \varphi + I' \cos \varphi. \]

When the \( \Sigma^0 \) is not observed, \( \theta \) and \( \varphi \) are unknown, but they must satisfy the following relation:
\[ u'' \cdot K = u'' \cdot K(1 + \kappa^2) + K \cdot \mathbf{I} \sin \cos \varphi + K \cdot I' \cos \theta, \]
due to the energy-momentum conservation in \( \Sigma^0 \rightarrow \Lambda^0 + \gamma \) decay. Note that \( \theta \) is an even function of \( \varphi \). When the \( \Sigma^0 \) is observed, the \( \Lambda^0 \) polarization is given by Eq. (53):
\[ \delta = -\eta K_2(\mathbf{d} \cdot \mathbf{u})u'' = -\eta(\theta)K_2(\mathbf{d} \cdot \mathbf{u})(-K_1u_1 + K_2u_2) = \delta(\varphi). \]

If the \( \Sigma^0 \) is not observed, the \( \Lambda^0 \) polarization is then:
\[ \delta = \langle \delta(\varphi) \rangle = -\frac{1}{2\pi} \int_{2\pi} \delta(\varphi) d\varphi. \]

We leave to the reader to compute:
\[ \langle \delta \rangle = -K K_2 \mathbf{u}_1 \left( \frac{1}{2\pi} \int_{2\pi} \eta(\theta) \sin^2 \varphi d\varphi. \right) \]
From argument of invariance under $P$, $T$ we could predict that $\langle 8 \rangle$ is transverse and orthogonal to the 3-plane $t, p_0, p_3$ as if the $\Lambda^0$ were produced directly by $\pi^- + p^+ \rightarrow \Lambda^+ + K^0$. [The proof of $\Sigma^0$ production is given by $\langle M^0t^0 - M(\pi) \rangle = M^{n0} + M^0 - 2 M M^0 \sin \theta \neq \mu K^0$.] (A.23) The asymmetry in $\Lambda^0$ decay even when the $\Sigma^0$ is not observed is therefore a measure of the function,

$$\frac{1}{2\pi} \int_0^{2\pi} \eta(\theta) \sin^2 \varphi d\varphi,$$

of the polarization $\eta$ of the $\Sigma^0$.

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**I. INTRODUCTION**

Measurements have been made, using scintillation counters, of the angular distributions of $\pi^-$ mesons scattered from carbon at 69.5 and 87.5 Mev and from oxygen at 87.5 Mev. The experimental work is an extension of that of Baker, Rainwater, and Williams\(^3\) (BRW), in which the scattering of 80-Mev $\pi^-$ mesons from Li, C, Al, and Cu was measured. In their experiment, scattered pion energy was determined from the range of pions stopped in a counter. This technique afforded considerable improvement in energy resolution over that obtained previously with counters\(^4\) and cloud chambers.\(^5\) The present experiment employed four such counters in succession, the “multi-counter,” to increase the data-taking rate. The energy resolution in either experiment was sufficient to separate out pure elastic scattering from all inelastic scattering for carbon and oxygen. In the case of lithium, BRW employ the electron scattering data\(^8\) to argue that the contribution of scattering from the first excited state to the measured elastic scattering is small. No other levels contribute.

Recent experiments have been performed by Kane,\(^9\) $\pi^+$ scattering from carbon at 31.5 Mev; and Fujii,\(^10\) 150-Mev $\pi^-$ scattering from C, Al, Cu, and Pb. Kane measured total pion energy by means of pulse height in a scintillation counter with an (absolute) energy resolution comparable to our own. Fujii measured quasi-elastic scattering into a 15-Mev interval by means of total energy determination in a Cerenkov counter but could not separate out pure elastic scattering.

Baker, Byfield, and Rainwater\(^11\) (BBR) have fitted optical model calculations to the data of BRW. The optical potential used was a modification of the one of Kisslinger.\(^12\) It removes a nonphysical divergence in the unmodified form. The potential includes a term in the gradient of the nuclear density which arises from the important $p$-wave contribution to the elemental $\pi$-nucleon scattering in the nucleus. Hence, the predictions are particularly sensitive to the nuclear edge thickness. The model gives good fits to the data at all angles and for nuclear radii consistent with the results

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\(^3\) R. E. Williams, J. Rainwater, and A. Pevsner, Phys. Rev. 101, 412 (1956).


\(^8\) J. F. Streib, Phys. Rev. 100, 1797 (1955).


\(^11\) W. F. Baker, H. Byfield, and J. Rainwater, Phys. Rev. 112, 1773 (1958). Equations (2), (13a), and (13b) are printed incorrectly in this paper. The sign of the right side of Eq. (2) should be reversed. The expressions for $\mathcal{C}$ and $\mathcal{C}'$ in (13a, b) should be divided by $A$. In the present paper $C \rightarrow C_1$ and $C' \rightarrow C_2$.

\(^12\) L. S. Kisslinger, Phys. Rev. 98, 761 (1955).